Electromagnetic Field Plot of an Inductive Window by the Moment Method

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Abstract —**A moment method is used to plot the electromagnetic field of an inductive window in a TE_{10}-mode rectangular waveguide. Green's dyadic functions are derived based on Tai's approach, which is a modified form of Hansen's vector wave functions. Based on the computed electric fields, the S matrix and the equivalent aperture reactance of the waveguide window are calculated. This calculation agrees with the previously published closed-form results of Marcuvitz.**

I. Introduction

THE objective of this paper is to show a plot of electromagnetic field of an inductive window obtained by the moment method. Waveguide windows are widely used in filter and impedance matching sections in rectangular waveguide systems. Although the impedance of this window has been investigated in the past, the electromagnetic fields in close proximity to the window have not been studied well [1]–[4]. To the authors' knowledge, no field plots are available. This paper proposes a method to obtain the field plots in a rectangular waveguide with a window using Green's functions [4]–[6]. Specifically, the total field near the window may be divided into two field contributions by the induction theorem [4]–[6]. These fields are the incoming field and the scattered field which is generated by an equivalent mathematical source which is used to represent the window.

II. Plot of Electric Fields

In Fig. 1, a conducting iris of $d \times b$ to form a waveguide window of $(a-d) \times b$ is illustrated.

A graphical representation of the incoming, scattered, and total fields in the waveguide was obtained by plotting the instantaneous fields at a given point in time as a function of position $\boldsymbol{r}$ in the domain of the waveguide.

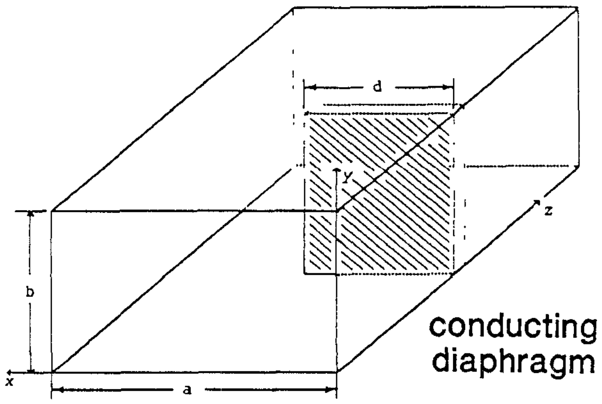

Fig. 1. Waveguide window in a section of rectangular waveguide.

The instantaneous fields in general are expressed by

$$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}\{\boldsymbol{E}(\boldsymbol{r})e^{j\omega t}\} \quad (1)$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \mathrm{Re}\{\boldsymbol{H}(\boldsymbol{r})e^{j\omega t}\}. \quad (2)$$

Only plots of the electric fields shall be shown in this paper.

Consider first the case with only the TE_{10} dominant mode propagating in the waveguide as the incoming field. The incoming electric field for this mode has only a y component. Since the incoming electric field $E_y^i(r,t)$ is independent of the position y, its magnitude is independent of y for $0 < y < b$. The amplitude H_0 of the longitudinal component of the incoming magnetic field was set to

$$H_0 = \frac{1}{\omega\mu}\frac{\pi}{a} \qquad (A/m) \quad (3)$$

such that the amplitude of E_y^i was normalized to 1 (V/m). The width, a, and the height, b, of the waveguide were chosen for X-band rectangular waveguide. The operating frequency, f, was chosen such that all modes were in cutoff except for TE_{10}. The waveguide wavelength, λ_g, was determined by common waveguide theory [1]–[4]. Calculations are performed using a VAX computer [8].

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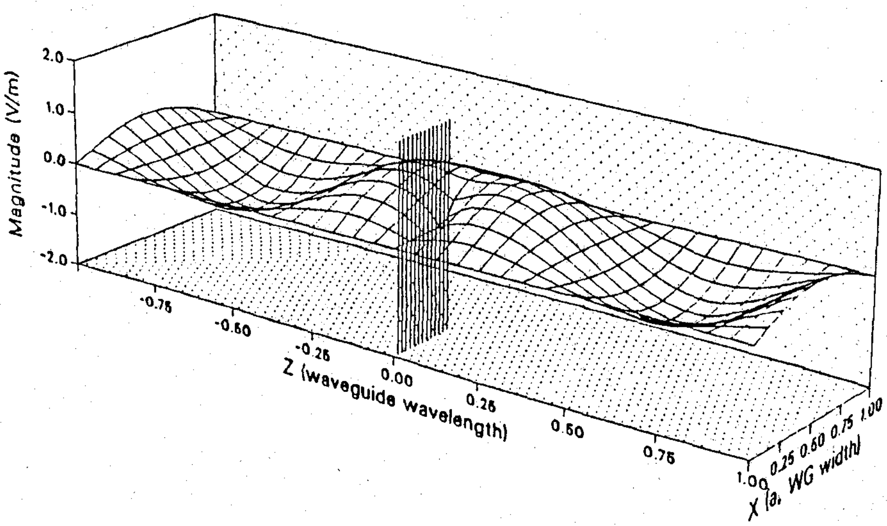

Fig. 2. Scattered electric field: $\omega t = 0.00\pi$, $f = 10.00$GHz, $\lambda_g = 4.87$ cm, $a = 1.905$ cm, $b = 0.952$ cm, diaphragm width = 0.762 cm.

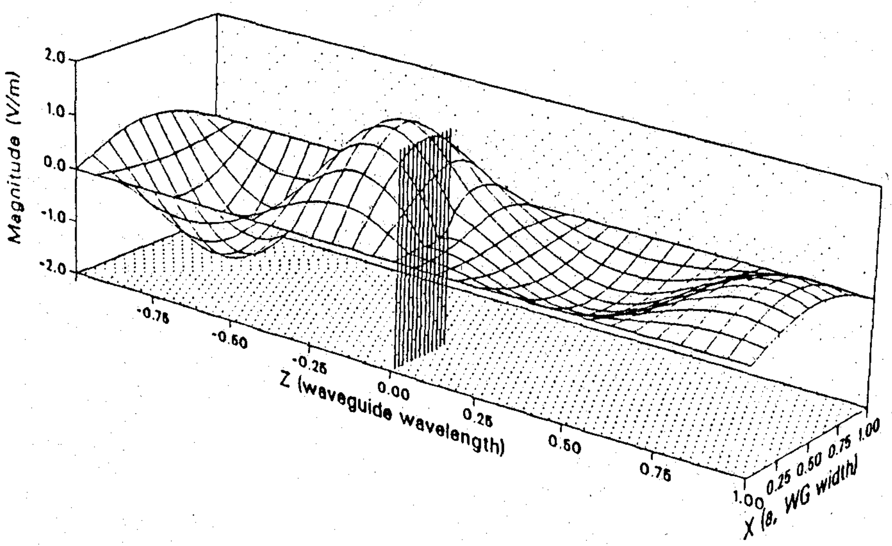

Fig. 3. Total electric field: $\omega t = 0.00\pi$, $f = 10.00$ GHz, $\lambda_g = 4.87$ cm, $a = 1.905$ cm, $b = 0.952$ cm, diaphragm width = 0.762 cm.

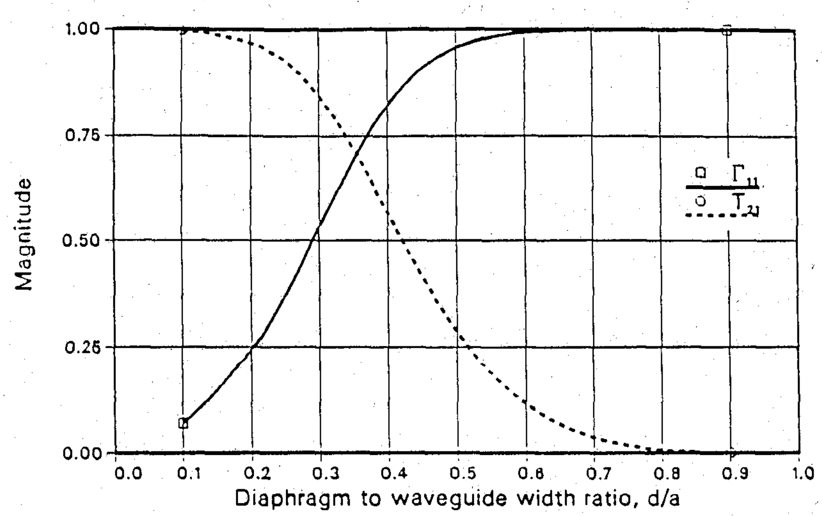

Fig. 4. Magnitude of Γ_{11} and T_{21} of the window: $a = 1.905$ cm, $f = 10.00$ GHz, $\lambda_g = 4.87$ cm.

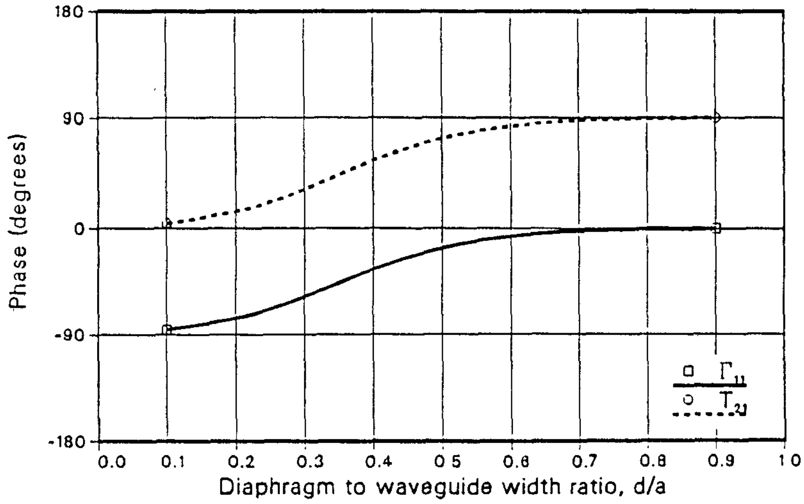

Fig. 5. Phase angle of Γ_{11} and T_{21} of the window: $a = 1.905$ cm, $f = 10.00$ GHz, $\lambda_g = 4.87$ cm.

The scattered electric field for the TE_{10} incident field is given by

$$E_y^s(r) = \sum_{i=1}^{N} \left\{ G_{iy}^y(r/r_i) \cdot \left[j\omega\mu I_y^i(r_i) - k_z \frac{\omega\mu a}{\pi} H_0 \sin\left[\frac{\pi}{a} x_i\right] e^{-jk_z Z_i} \right] \right\} u_y. \quad (4)$$

The Green's function, $G_{iy}^y(r|r_i)$, is given by

$$G_{iy}^y(r/r_i) = \frac{j}{ab} \sum_{m,n} \left\{ \frac{2-\delta_0}{k_L^2 k_z} \left[m\frac{\pi}{a} \right]^2 - \frac{k_z^2}{k^2} \left(\frac{n\pi}{b} \right) \right]^2 \cdot \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sin\frac{m\pi x_i}{a} \cos\frac{n\pi y_i}{b} e^{-jk_z|z-z_i|} \right\} \quad (5)$$

where $k^2 = \omega^2 \epsilon\mu$. The infinite summation over m must be truncated. For $m > 1$, the scattered field is in cutoff and is an evanescent field which is localized at r', being attenuated as a function of $e^{-k_z|z-z'|}$ from the window. The amplitude of Green's function of each mode of this evanescent field is, from (5),

$$A_{iy}^y(m) = \frac{2}{ab\sqrt{\left[\frac{m\pi}{a}\right]^2 - k^2}} \quad (6)$$

which decreases as m increases and converges to zero as m approaches infinity. The summation of A_{iy}^y over m will approach infinity as m approaches infinity. To determine the number of modes at which to truncate, the percent difference between the m and $m-1$ amplitude of the summation was determined by choosing the value of $m = 197$, giving a percent difference of 0.1.

The coefficients $I_y^i(r_i)$ of the total current were calculated as follows:

$$[I_y^i] = [G_{iy}^y]^{-1}[E_y^i]. \quad (7)$$

The instantaneous scattered electric field, $E_y^s(r,t)$, was plotted for $\omega t = 0$, as shown in Fig. 2. The instantaneous total electric field, $E_y(r,t)$, was plotted for the same ωt, as shown in Fig. 3. The discrete set of lines in the figures indicate the position r' of the sources and thus the diaphragm position in the waveguide. The number N of current filaments chosen to represent the impressed sources and their positions is also indicated by these lines. For this case, $N = 13$ and the width, d, of the diaphragm was $0.4a$, as shown. The magnitude scale was increased to 1.7 (V/m) compared with a maximum magnitude of 1.0 (V/m) for the incoming electric field.

The plot of the total field shows that the wave reflected from the diaphragm, as shown in Fig. 3 for $z < 0$, is in phase to some degree with the incoming field such that the magnitude of the total field is greater than 1.0 (V/m).

A. Scattering Matrix of the Window [1], [2], [4]

The following scattering coefficients can be determined:

$$S_{22} = S_{11} = \Gamma_{11} = \left.\frac{E_y^s(r)}{E_y^i(r)}\right|_{z=P_1} \quad (8)$$

$$S_{12} = S_{21} = T_{21} = \frac{[E_y^i(r) + E_y^s(r)]}{E_y^i(r)|_{z=P_1}} \quad (9)$$

where Γ_{11} is the reflection coefficient at port 1 at $z = P_1$; T_{21} is the transmission coefficient from $z = P_1$ to port 2 at $z = P_2$ of the two-port structure of this window; and $E_y^i(r)$ is given by

$$E_y^i(r) = -j\omega\mu \frac{a}{\pi} H_0 \sin\frac{\pi x}{a} e^{-jk_z z} u_y. \quad (10)$$

$E_y^s(r)$ is given by (4). The reference planes P_1 and P_2 were chosen to be several integral number of waveguide wavelengths, $n\lambda_g$, away from the plane $z = z'$, where the window is located. The planes must be far enough from the window that the magnitudes of the evanescent scattered fields are near zero.

Both S_{11} and S_{21} are calculated as functions of the diaphragm to waveguide width ratio, d/a, for $0.1 <$

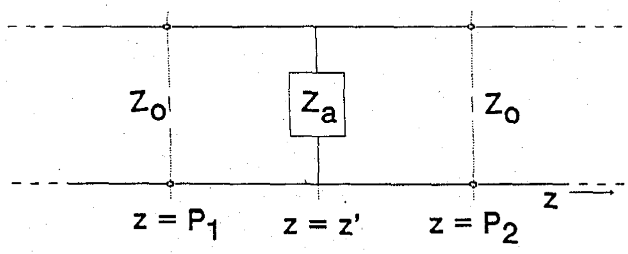

Fig. 6. Equivalent circuit of a window with zero thickness.

$d/a < 0.9$. The magnitude plots are shown in Fig. 4 and the phase plots are shown in Fig. 5. The modal summation was over 54 modes, and N was varied with d/a, $5 \leqslant N \leqslant 29$. The reference planes P_1 and P_2 were set at λ_g from the diaphragm position.

B. Comparison with Known Results [1]–[3]

The window as a discontinuity in rectangular waveguide was analyzed and compared with measured results in Marcuvitz [1]. An equivalent circuit of the window was assumed by Marcuvitz as shown in Fig. 6 and has analysis provided a closed-form approximation to the impedance Z_a of the equivalent circuit using an equivalent static method [1].

The impedance Z_a in Fig. 6 is purely reactive since a lossless network was assumed;

$$Z_a = jX_a. \tag{11}$$

The impedance Z_a normalized to Z_0 can be determined from the S matrix as follows [6]:

$$\frac{Z_a}{Z_0} = \frac{2T_{21}}{(1-\Gamma_{11})^2 - (T_{21})^2} \tag{12}$$

where Γ_{11} and T_{21} are given by (8) and (9). The characteristic impedance, Z_0, does not need to be determined since the reactance given in Marcuvitz [1] was normalized as $X_a \lambda_g / (aZ_0)$. This normalized reactance is plotted in Fig. 7 as a function of the diaphragm to waveguide width ratio, d/a, for $0.1 < d/a < 0.9$. The results from Marcuvitz's closed-form solution are also plotted in Fig. 7 for comparison [1, p. 224]. The modal summation was over 54 modes, and N was again varied[1] with d/a such that $5 \leqslant N \leqslant 29$ and for the width of the diaphragm of the window in the range $0.1a < d < 0.9a$.

III. Conclusions

This paper has presented a method of moments applied for producing the electromagnetic field plot in a rectangular waveguide with a window. From these plots, it has been shown that the scattering coefficients and normalized impedance of this structure are obtainable.

The results of this method agree well with published data for the analysis of a window in waveguide for the TE_{10} mode propagation and for the width of the diaphragm of the window in the range $0.1a < d < 0.9a$.

[1]Computer programs are available from the authors.

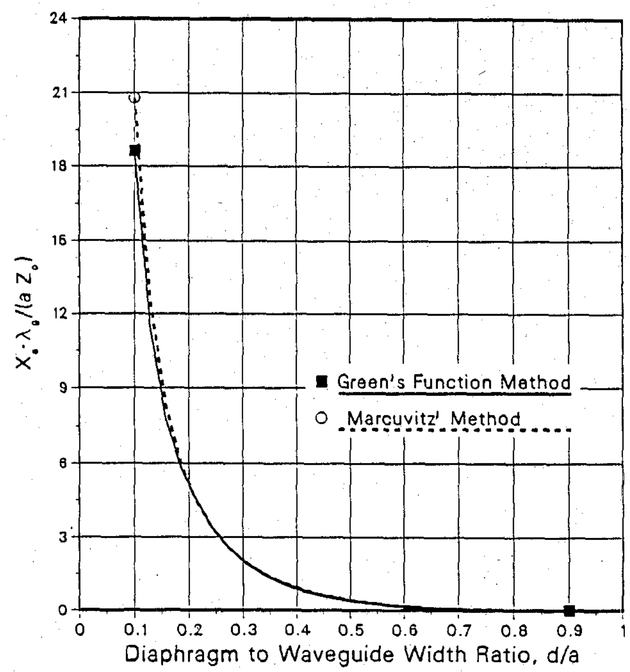

Fig. 7. Shunt reactance X_a of the window: $a = 1.905$ cm, $f = 10.00$ GHz, $\lambda_g = 4.87$ cm.

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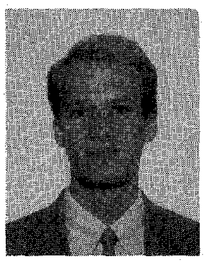

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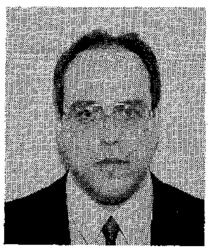

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